Final Projects from the Illinois Math Badging Initiative Micro-Internship

July 2024
**EdSystems**

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EdSystems fosters collaborative partnerships to design, implement, and evaluate policies and practices that ensure successful transitions to and through postsecondary and career opportunities for students, with a particular emphasis on historically marginalized populations.

**VISION**
EdSystems helps create a world where students have clear, unambiguous paths to college and career opportunities that equip them for meaningful participation in the global economy.

**ACKNOWLEDGEMENTS**
EdSystems would like to thank XQ Institute for its support of the Illinois Math Badging Initiative and its micro-internship.

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Introduction

Illinois Math Badging Initiative

The Illinois Math Badging Initiative (IMBI) is an initiative to develop an Illinois high school math badging system through the partnership of EdSystems, XQ Institute, and Student Achievement Partners.

The math badging system serves as an alternative credentialing mechanism aligned to Illinois Learning Standards and transitional math competencies. Badges allow students to certify learning acquired through various sources, such as coursework, independent study, summer school, and work-based learning. The badges can translate to credit for transitional math, high school math courses, and early college credit.

The initiative integrates with EdSystems’ broader efforts to advance racial equity by prioritizing the elimination of disparities in education and employment outcomes for young people from historically marginalized populations.

Micro-Internship

Aligned with IMBI’s goals, EdSystems, together with the XQ Institute, conducted a 6-week, 30-hour, paid virtual micro-internship for 11 high school students in spring 2024. The interns explored real-world applications of math, developed recommendations for integrating math into various activities, and connected their findings to career interests and classroom learning.

Interns from the 2024 pilot cohort produced written reports detailing insights they gathered on capturing math outside the traditional classroom. Interns were also asked to connect their presented topics and skills to classroom learning and choose a math badge or badges that best related to their final project topics. Key insights from their final reports, as well as their full reports, are shared below.
Key Themes of the Micro-Internship Final Projects

Each week, students researched and presented on a wide variety of topics and skills. Over the span of the internship, students began noticing common themes between their own weekly presentations and those of their peers, despite presenting on different topics and skills. These themes, present across the student reports shared in this document, are summarized below:

Math in Everyday Life

• Math is ubiquitous, influencing daily activities, from complex tasks like developing algorithms or discovering medications to simpler tasks such as budgeting or sports scoring. Recognizing its presence helps demystify math and shows its practicality.
• Real-life application of math, such as using percentages, ratios, fractions, and geometry, helps students understand its importance beyond the classroom.

Interdisciplinary Learning

• Combining math with other subjects, like history or science, can make learning more engaging and relevant. For example, studying the geometry of ancient pyramids or the physics of motion through vectors.
• Journaling about personal interests, such as music or finance, and relating them to mathematical concepts fosters a deeper connection and enthusiasm for the subject.

Real-World Problem Solving

• Using real-life scenarios to teach mathematical concepts helps students see the relevance and utility of their learning. Examples include calculating travel distances, managing finances, or understanding data trends in the stock market.
• Hands-on projects and experiments, like measuring the properties of shapes or calculating the distance traveled by a moving object, reinforce theoretical knowledge through practical application.

Mathematical Concepts in Various Contexts

• Introducing concepts like linear equations, the Pythagorean theorem, and vectors through relatable examples helps students understand their importance and application.
• Demonstrating how math is used in different fields, such as sports, finance, and science, shows the versatility and necessity of mathematical knowledge.

Student-Centered Learning

• Encouraging students to explore topics that interest them and relate those interests to mathematical concepts makes learning more personalized and engaging.
• Providing opportunities for peer interaction and discussion about how math is encountered in daily life fosters a collaborative learning environment.
Project Examples
An Analysis of Math Outside the Classroom

by Aarav Shah, Illinois Mathematics and Science Academy

The question “When am I ever going to use this?” echoes through countless math classrooms each day. For many students, the answer might be “not directly.” Solving a system of equations or finding the derivative of a function may not be a regular occurrence in their chosen careers. However, math is far more than a collection of formulas and procedures to be memorized and repeated. It’s a powerful way of thinking that equips students with valuable, transferable skills. While practical applications of math are undeniable, its true strength lies in fostering critical problem-solving, logical reasoning, and analytical thinking – skills that empower success in any field.

The influence of math extends far beyond the classroom, shaping the world around us. From the calculations behind our electronic devices to the engineering feats that create architectural marvels, the majority of people’s interactions rely on a foundation of mathematical principles. For example, binary code, algorithms powering search engines, and even touchscreens and motion sensors all stem from mathematics. Even the seemingly mundane act of texting a friend or swiping through photos involves hundreds and thousands of complex mathematical operations happening behind the scenes. Ultimately, regardless of profession, we all utilize math every day through these tools that have become extensions of ourselves.

Just as computers rely on algorithms – a set of precise, logical instructions – to solve problems, math equips us with the same ability. By taking apart mathematical puzzles, identifying patterns, and systematically testing solutions, people learn to approach challenges with a logical framework. It teaches us to identify patterns, analyze relationships between variables, and construct sound arguments based on evidence. These are all skills that translate to real world scenarios. When faced with difficult decisions, whether it’s troubleshooting a technological issue or navigating a personal conflict, the logical reasoning honed through math allows us to analyze the situation, evaluate options, and arrive at well-considered solutions. Math is a discipline that cultivates the skills necessary to navigate life.

Math is not just about finding the right answer; it’s about the process of getting there. Every mathematical problem presents a challenge, requiring us to break it down into smaller, more manageable steps. We learn to identify the problem’s core elements, explore different strategies, and rigorously test our solutions. This systematic approach translates into a powerful problem-solving ability. Whether troubleshooting a technical issue at work, tackling a personal challenge, or devising a plan to achieve a goal, the ability to break down problems, analyze options, and implement solutions effectively is a universally applicable skill.

While often perceived as a rigid field, math can surprisingly spark creativity and innovation. Advanced mathematical concepts can lead to unexpected connections, inspiring new ideas and solutions. The very act of solving a challenging math problem often necessitates thinking outside the box, exploring unconventional approaches, and embracing unconventional solutions. The ability to break away from established patterns and challenge assumptions is a clear indicator of creativity. Additionally, mathematicians themselves are often driven by a sense of curiosity and a desire to explore the unknown. Whether it’s a scientist conducting groundbreaking research, an artist pushing the boundaries of their medium, or an entrepreneur developing a revolutionary product, the spark of creativity often ignites from a foundation of questioning and a willingness to explore the unconventional – skills nurtured by mathematics.

Furthermore, math creates a growth mindset, which is a crucial life skill. It encourages people to view their failures not as setbacks, but as stepping stones. Math problems often present challenges that require perseverance and a willingness to learn from mistakes. The process of working with a complex question, exploring different approaches, messing up multiple times, and finally finding a solution instills a sense of accomplishment and the confidence to tackle even more challenging problems in the future.
This journey through problem-solving cultivates the belief that intellectual abilities can be developed through effort and dedication. In addition, math celebrates the beauty of multiple solutions to a single problem. Students learn that there are often various paths to reach the correct answer, encouraging them to explore diverse approaches and embrace different styles of thinking. An openness to new ideas and the understanding that mistakes are stepping stones to learning are attributes of a growth mindset, challenging students to approach challenges with a confident “yet” rather than a limiting “never.” By nurturing intellectual curiosity and a willingness to persevere, math equips us with the tools and mindset necessary for lifelong learning and success.

The benefits of a strong mathematical foundation extend far beyond the individual. When a large portion of a population possesses critical thinking, problem-solving, and analytical skills fostered by math, it has a ripple effect on society as a whole. In an age where information is easily accessible, the ability to analyze and interpret data effectively is no longer a luxury, but a necessity.

Indicators such as Math Badging Initiative M212, specifically focused on mathematical modeling and data analysis, empower individuals to become active participants in this information age. M212 recognizes the crucial role of math in dissecting complex datasets. Through badge-based learning, students can develop the skills to identify trends and patterns hidden within mountains of data. They can use mathematical modeling to analyze social media data, predict economic trends through statistical analysis, or even understand weather patterns through the lens of mathematics. M212 equips students with the tools to draw meaningful conclusions from the ocean of information at our fingertips.

Additionally, on a societal level, strong mathematical literacy allows for informed policy decisions. Governments can leverage mathematical models to predict climate trends, optimize resource allocation, and develop effective solutions to social challenges. Scientific advancements too, rely heavily on the power of mathematics. From simulations and statistical analysis to the very foundation of physics and engineering principles, math enables us to understand the world around us and push the boundaries of human knowledge. In this information age, a society with a strong grasp of mathematical concepts is not just better equipped to navigate the complexities of the modern world, but also holds the key to progress and innovation for generations to come.

Clearly, mathematical experiences employ us with a powerful way of thinking, applicable far beyond the classroom. They foster problem-solving, logical reasoning, analytical thinking, and a growth mindset, all skills that empower success in any career and throughout life. Initiatives like the Illinois Math Badging Initiative allow students to learn and explore on their own, deepening their understanding and cultivating a love of learning. Ultimately, math is not just about finding the answer, it’s about the journey of discovery, the process of grappling with challenges, and the confidence to approach life with a logical and creative mind. Math, by its very nature, ignites curiosity and equips us with the tools to explore the world around us, not just with formulas, but with a sense of wonder and a thirst for knowledge.
Utilizing Math Outside of The Classroom

by Rachel Ruth Bumatayo, Charleston High School

Students often have a common question when they work on math in their classrooms. They question when they will need to use a specific math skill in life. The way that math is presented and taught in the classroom oftentimes does not explicitly show why students may need this skill later in their lives. The truth is, math is everywhere in this world and serves as a foundation for many fields of work and hobbies a person may be involved in.

While a student may not always be using complex math that they learn in class, the basics of math are extremely important. Addition, subtraction, multiplication, and division are all basic arithmetic, and because of how basic it is, a person might completely overlook the fact that math is still being used in their everyday life.

The math work that is assigned to students often gives them an unrealistic scenario they cannot relate to. For example, the most common elementary grade question that many people have come across is where the person in the math problem buys an extremely large quantity of a certain fruit and the student must solve for the total price of the fruit when they are given how much a single fruit costs. This is not a relatable and realistic scenario, so it may be difficult for students to understand where and when this kind of math will be applied, especially for younger kids.

If a teacher gives their students a chance to investigate how they can apply math to areas that interest them and credit the students for their investigations, then this could help students understand and enjoy math even more. This can all happen outside of the classroom. Younger kids will be able to understand why it is important to learn math and it can help older kids, such as high schoolers, to figure out what field of work they want to go into based on how difficult the math may be. So, how exactly does a student apply math in their everyday lives?

Types of Math That Can Be Applied Outside of the Classroom

Solving Linear Expressions, Equations and Inequalities (Badge 101.b)

It may be difficult to understand how linear expressions, equations, and inequalities can be used in a person's everyday life, but when people are managing their finances and budgeting, it can be helpful to use one of these equations or expressions that a student would typically use in a math class. For example, a family might be on vacation and needs a taxi to come pick them up. If the taxi has a flat rate of $10 and charges $4 for every mile, then the equation that might be used in this situation will be: \( y = 4x + 10 \).

Many students also have a job that they balance with school in order to prepare funds for college and to be able to spend their own money freely. In this situation, pretend a student would like to buy an item that costs $120 and wants to know what number of hours they could work to have $120 or more dollars. The student receives a bonus of $10 and their wage is $5 an hour. The equation used in this situation would look like this: \( 120 = 5x + 10 \). Companies use linear inequalities for their finances and for budgeting. However, not everyone needs to be running a large company in order to use these equations. Some students even run their own small businesses, such as selling candles, jewelry or photography.

Students that run their own business should also utilize linear inequalities to help them manage their finances and budget properly. In this scenario, a small bakery needs to buy some flour but wants to keep their budget of $200 dollars in mind. One pound is $5, but they already spent $145. The inequality will be written as: \( 200 \geq 5x + 145 \). These scenarios are more likely to happen and are relatable for students, so this situation may look like a possible way for students to earn math credit when traveling, working, and budgeting. Since linear expressions and equations can be drawn as graphs, it makes it extremely beneficial to use these math skills in finance, since a student can get a quick and easy view on how they
are doing financially by using the graph. Alongside earning credits for their math class, this will teach students how to responsibly manage their money and will prepare them for the future.

This badge is also similar to Badge 101.a, which is titled “Reasoning About One-Variable Equations and Inequalities”. This badge can help students understand what number is unknown in that equation and where it will be in the equation as well. It also requires students to write about such equations that they will see in the real world, so this can help them to prepare for when they will need to use the linear equation, expression, or inequality later in life.

Comparing Different Data Sets (Badge 111.f)

When comparing different data sets in the classroom, it may seem more realistic, as a student might be asked to compare how many books were bought from two different bookstores. But, comparing data sets is extremely useful in the real world as well, especially for students that run businesses, or for students in clubs, if the club is planning to sell things. This badge requires students to create two data sets that can be compared to one another, find the center and variability for each of the two, compare the data sets, then explain how they compared the two and why they chose to do it that certain way. How can one put that into practice?

For example, the art club may be holding a yearly art fundraiser and will be selling some artwork to raise money. However, they are not sure what to sell, so they decide to gather some information from a couple fundraisers from the years prior. These students can calculate how much of each type of art was sold, such as pots, paintings, collages, etc. Then, the students can calculate how much each type of art was sold on average each day, compare the results, and figure out which type of art should be displayed more at the fundraiser in order to accumulate more money. Comparing data sets can help students make decisions once they have solid evidence that one variable has more favor than the others. This badge also ties into Badge 113.f, which is titled, “Using Probabilities to Make Decisions”. If the art club can conclude the probability of a person buying a certain pot based on the previous data sets, then they can also figure which art item should be made the most in order to maximize their profits. When the students have both the graph for a visual, and a percentage, it can reinforce a decision that a student will make.

Students may also create and compare data sets for non-financial situations as well. For example, if a senior high school student wanted to hire a local photography business for their senior pictures, but wanted reviews from people about two potential photography businesses to hire. The student will create some sort of visual data, then will have to find the average, or the mean, for the reviews of the two different businesses.

Conclusion

The situations above were mostly financial situations, since no matter what field a person may work in, being able to properly manage finances and budget properly are all extremely important skills in life that require math. Of course, a student can apply math outside of financial situations, but again, finance is super important, and it shows how any student can identify when they use math easily since money is used in our everyday lives. Since everyone has such different lives and interests, letting students come back and present their investigations with others can help students to teach others about topics that interest them, and allowing students to see a new perspective on a hobby they may have never even thought of before. Allowing students to earn math credit by investigating how math is used outside of the classroom can help students understand why math is important and how specific types of math can be used in certain real life situations. While only a few math topics were covered very briefly, they are used everywhere, and even the most complex math has its use in life.
Math is used constantly everyday throughout people's lives and they don't even realize it. It's the building block of life and everything in the world depends upon it. Anything from coming up with very precise and complex computer algorithms or discovering world changing medications involve math. Math even exists within nature surrounding almost everything that has ever existed. There is symmetry and patterns within wildlife. Society loves anything to do with patterns or symmetry. It's just how the human brain was developed. Without math the world would end and create chaos in the human population.

Most people believe math can only relate to the classroom and people don't use it outside of school. However, they are completely wrong. In school, the student population is taught how to make graphs and create or identify shapes from a graph. The teachers teach students math correlating to graphs by using equations that relate to real world problems. It allows students to make the connection to the topic getting taught to them while enhancing their skills later on. It helps to prepare students for situations in the world where they encounter the same skills they were previously taught. Students get taught percentages, ratios, fractions, probability, geometry, statistics, calculus, exponents and their rules, etc. Most of the math students learn actually benefits them outside of the classroom.

Teachers have to be able to teach their curriculum in different ways to benefit students who learn differently from their peers. Everyone learns skills in different ways and at their own pace. It's important for teachers to find which ways of teaching works best for most students to succeed. They also need to consider the age level of the students. Young kids learn about buying a certain number of food from a store and either add or subtract how much food someone ate from the pile of food to find the total amount of food left. The most important thing teachers want their students to learn is arithmetic math. Younger kids’ math typically doesn't give a realistic real world problem. As students get older it starts to slowly become more realistic. Pre-teens have to find the distance they can travel in a specific amount of time while finding how many times someone would have to fill up their vehicle before reaching their destination. This situation is a more realistic world problem. High school math is the most realistic to real world problems when talking geometry, fractions, statistics, and percentages. Most of society uses all of those math skills everyday. A struggle teachers must constantly fight with students is trying to keep scholars focused on what is getting taught. Young kids are pretty easy to keep entertained but as people get older they start to get distracted more easily. They also don't care about what is being taught because most often students have a mindset that this math will never be beneficial in the real world. If teachers can shift the way adolescents think and show them how their curriculum relates to real world situations, then students would feel more motivated to learn and could strengthen their skills.

Math is used outside of the classroom in a variety of ways. People use math in their daily lives, workplace, and traveling to their job. Anything from figuring out how many times someone can drive to and from work before refilling their car with gas, calculating how much money it takes to buy groceries, working at NASA using complicated equations that revolve around physics, etc. all require math. This is why math is so vital for the human species to survive. None of the problems humans use math for could have existed without the Sumerians, who are believed to have created the first math systems and math tables. It's very interesting how society started with basic arithmetic and, as the years went on, started to create complex equations that society uses today, all based around the first math systems. It all started with a generation creating math and teaching their generation about it. After the first generation dies the second generation critiques what the previous generation discovered and teaches the new generation what they discovered. It's an ongoing cycle that has existed and worked for thousands of years.

**Linear Expressions, Equations, and Inequalities (Badge 101.b)**

Some people may not understand how linear expressions, equations, and inequalities are used daily. Students typically use them in a classroom setting but how can people who don't attend school use
Managers of stores and businesses use these skills when they are budgeting or dealing with finances. For example, a store hires 6 workers and each worker gets paid $15 an hour. The monthly cost of fixed expenses is $250. The store has a salary of $1,800 per month. What is the most amount of hours that a worker can work in a month without letting the store go into debt? An equation to represent this could be $1,800 \geq 6 \times 15h + 250$. Being able to write and solve equations that apply to real world problems is important. Managers constantly have to do this to keep their business out of debt. Basketball and other sports use these skills when trying to find the total number of points an individual scored in a game. For example, Billy made seven three point shots, four regular shots, and eight free throws. How many points did Billy make? An equation to represent this could be $P = 7x + 4y + 8z$. Knowing how to solve equations is very beneficial for the real world. Many athletes constantly figure up their scores for their sports and they do it without even realizing an equation is used to solve the problem.

**Using Geometric Shapes To Describe Objects (Badge 151.b)**

Many people don’t realize when they describe an object to someone it classifies as a geometric shape. For example, if someone says they are bringing a cake to an event, most people would have a general idea of what shape of cake to expect. A typical cake is either a circle or a rectangle. People know a cake is circular due to the shape having no lines. If someone cut a line through the diameter or center of a circle there would be two equal sized semicircles. A rectangle is easy to identify based on the two sets of opposite parallel sides. The sides of a rectangle are shorter than the top and bottom. If someone cut a diagonal line through the center, the lines would bisect each other. There would be four tiny little triangles leftover. Knowing properties of shapes makes it easier to describe an object to someone. For example, someone goes shopping and is looking for an item. They can’t find it and ask an employee by describing the item’s name and shape. The employee is then able to direct the customer to the right place based on what the shape is and how the item looks. It’s important to be able to describe shapes and know the different properties of shapes.

**Right Triangles and Pythagorean Theorem (Badge 151.h)**

Most people don’t think the pythagorean theorem is very useful but they are wrong. It’s used quite often and helps when constructing objects. For example, someone has to create a massive food sculpture in a baking competition but doesn’t know how to find the hypotenuse of a triangle. This is a great real life situation of knowing how to solve for pythagorean theorem. The contestant is making a giant slice of pizza with the adjacent side of the crust being twenty-five inches long and the opposite side of the pizza being thirty inches long. The contestant is now able to use the pythagorean theorem to solve for the missing side of the pizza. The equation would be set up as $25^2 + 30^2 = c^2$. The contestant solves the equation and finds that the missing length of the pizza is thirty-nine inches. The contestant realizes how important the pythagorean theorem actually is and why teachers taught it to their students. The pythagorean theorem can also be used in sports. For example, Milly is working on her free throws and can’t figure out why her shot won’t go in. The height of the basket is twelve inches and the distance from the ground of the basket to her is sixteen inches. She can use the pythagorean theorem to solve for the length she needs to follow when angling her arm to shoot. The equation would be set up as $12^2 + 16^2 = c^2$. Milly solves the equation and realizes that the distance she needs to follow when shooting is twenty inches. She is thankful for her teacher teaching her this very important skill.

Math is a universal skill people develop and it unites everyone in the world. Teaching adolescents how to think about math outside of the classroom setting instead of the standard classroom setting is pretty important. Shifting the way of how kids think can create a generational evolution throughout the world. It will set kids up for success and allow society to grow stronger mathematically to solve bigger world problems compared to this day and age.
Mathematics All Around Us

by Lois A. Baker, Charleston High School

Students learn math by connecting it to the real world around them. The function of math is to assist people in their everyday lives, to better the world and create order and reason. Students have a hard time understanding concepts that they can not physically use in real life. Not being able to imagine the math you are learning can cause students to not fully grasp a concept. Relating math education to real life situations can better the population's understanding of mathematical concepts and enrich youths to be more engaged in the classroom.

Identifying Math

Building an understanding of the world strengthens students’ mathematical skills. Math is the foundation of all logic and reason that humans rely on for comprehension and development. From finances, music, and atomic structure math makes up everything that people know. Without mathematical reasoning and logic skills society would not have been able to advance to what it is today. Students should learn to identify math in the world around them to strengthen the way they view math in the classroom. By relating math to something that is familiar and visual, students can see how the math is represented and can better understand what is going on.

People learn in varying ways. The way one student learns might not work for another student. It is important that students consider their learning abilities and what makes most sense to them. And educators need to pick worksheet problems, assignments, and projects in a style that is suitable for all. The most achievable way to relate math to a majority is to implement real-life situations and concepts that students can find interest in. Keeping students interested is essential because teenagers in general do not find school work appealing.

Students can find math in their everyday lives in ways similar to which students in this internship did. Young students could be encouraged to have discussions with other students and teachers about how they saw math over the weekend or throughout the day. This method would also encourage peer interactions and share the opinions and daily lives of students. Hearing about how other students’ thoughts on math could inspire other students to think of math in a new way.

Using Math

Students can use real world situations to complete math badges and excel in their classes. There are many badges that relate to the real-life situations students have journaled about. There has been a wide variety of topics that people have discussed in their journals. Many looked into finance, chess, computer science and stocks. Or they focused on the arts, in things like photography, music, cooking, traveling. Interns picked what topic interested them the most, making the journaling process enjoyable and entertaining. These journals and real life situations relate to the math badges and how students learn them.

Focussing on one student’s journal, this intern chose to write about the calculations in the stock market. This intern discussed the formula to determine the relative strength index, as well as percentages of the total capital. Topics of this kind can relate to the learning badges like 100.d and 100.e, which pertain to data analysis and comprehension. Educators can teach topics such as these with examples like that stock market because the stock market is a physical display of data. Data in the stock market is physically represented by the lines and the direction they go, up for gaining, down for positive. Students could analyze the data in the stock market to reason which one would be a better choice to invest in. They could compare and contrast the data represented by the graphs, to see what stock has the better chance of going up (100.d). Seeing the data represented by a physical line could help young learners visualize addition and subtraction in a graph. High schoolers will learn financial competency and how to reasonably make financial decisions. Examples like this could help interpret proportional relationships and data displays.
Math is a subject that can be combined with other classes, like history. Including history in math classes can spark the interest of other students, appealing to a wider range of students’ hobbies. Learning the history behind the discovery of math can strengthen the population's understanding and context of how math functions. Ancient Egyptians only used math for practical purposes, such as rationing food. When looking at how they created the pyramids, it is purely coincidental that they’re pyramids were right triangles. An intern came up with an example worksheet involving Ancient Egyptian architecture and their way of making them. The worksheet had students create a pyramid with blocks, then students would measure their pyramid and see if it is a right triangle. This worksheet had students be hands on with creating their pyramid 3D. Then it encourages students to think of the real life consequences for their architectural decisions. This project would be a suitable final for badge 102.a (engaging in the modeling cycle) because young learners use their problem solving skills to create a 3D pyramid, then they use their mathematical skills to see what type of triangle they created. This has students test out their modeling cycle because they are figuring out how to create a pyramid through trial and error, as well as using math to prove their figure correct.

Some interns even conducted their own statistical research for their journaling skills. While writing about the correlation between music and math, a young student wanted to see if musical students excelled in math classes. This intern used a social media poll to ask students what math class they were taking, and what they thought their competency was in that class. The researcher found that students involved in band said they had a math competency of 82.86% when averaged. Students involved in choir and musicals had an average understanding of 77.14%. This interns’ journal statistics engaged them in averaging and collecting data. While this intern chose to use a table, you could also have plotted this data and used it to create a line of best fit. Collecting data like this students can provide evidence to check off badges like 111.b, 111.c, 111.d, and 111.e. All of these badges pertain to collecting and analyzing data. Educators could have an assignment or unit where students choose a question or two, and collect data based on their peers. This encourages peer interactors and learning about fellow teens. Young learners also get to choose a question that interests them, making the work more enjoyable and interesting. Assignments such as this engage students in asking questions and finding evidence to support it, as well as averaging data, plotting data, and analyzing data.

**Benefits**

When students are interested in what they are learning, they are more likely to pay attention. While journaling for the internship, the interns picked topics that they had a large interest in. This made researching and talking about the topics an enjoyable process for the writers. The interns spoke with enthusiasm and excitement about their journals and did a large amount of research as well. It was apparent that the interns had a nice time writing and gathering information when it was a topic of their choice. If the same option were to be offered to students, they would also have freedom to enjoy the work they are doing. Relating math to a large range of topics can help students engage with the math and find more joy and interest in what they have to learn.

Making work interesting and realistic for students encourages them to study and try hard on their work. Teens often ask “when will I ever use this in the real world?” and that is a valid question. By relating math to common situations or ideas educators help young learners understand abstract ideas and make the learning of math concepts have a true purpose. When students know how to use math in real life, they can pay more attention and absorb the information because they know they have a use for it. Finding the motivation to learn starts with having a purpose for knowing the information.
Using Physical Experiment in Inquiry-Based Alternative Mathematics Learning

by Karthik Prasad, Illinois Mathematics and Science Academy

Mathematics is a barrier towards many students’ success in STEM fields, particularly for those in under-represented communities. Many students struggle to demonstrate math competency in the classroom and do not learn well in the classroom, but could succeed in alternative math learning opportunities. The Illinois Math Badging Initiative (IMBI) aims to correct this via providing alternate ways of obtaining math credentials via Math Badges. [1] One way to do this is via physical learning, where students learn concepts via physically experimenting with real-world objects and phenomena. The benefit of this approach is that it also very easily ties together with inquiry-based mathematics, a learning method where students are guided through inquiring and conjecturing their own mathematical results. [2] This report overlies the general methodology and reasoning behind such an approach, and sample activities where it could be applied.

Physical or kinesthetic learning is a learning method where students learn via hands-on experiment with the ideas they are interacting with. For example, a kinesthetic approach to explaining how wind turbines work would be to have students build a physical wind-turbine step by step and work through which parts need to go where and why. [2] Many students learn better via a physical approach, and so it is worth exploring this as an option for the IMBI program, which aims to provide alternative methods for students to demonstrate competency. Students who do learn better via these physical tools can use them to learn concepts and demonstrate that they understand mathematics in ways they cannot when restricted to the traditional classroom environment. However, mathematics does not yet have a significant curriculum developed involving kinesthetic learning—the National Math Foundation (a premier mathematics curriculum center) reports only now that they are beginning to develop resources for kinesthetic learning [3]. So, for the remainder of this report, we discuss various methods of kinesthetic learning as applied to aspects in math learning.

To start, we discuss an elementary school level topic - the relation distance equals rate times time, or D=RT. D=RT is one of the most fundamental relationships in elementary school mathematics—every person in the world, after all, should know how rates work. But, nonetheless, it remains a challenging concept to interpret and utilize by students, and this is likely due to the fact that it is not sufficiently understood physically by students. The easiest way to visualize this relationship is to think, of course, with physics and time relationships. [4]

If someone is going 60 miles per hour, how far do they go in one hour? The answer is simple, 60. If you need to push a button 10 times a minute, how many times do you push a button per minute? 10 times. Students should physically play with these tools by doing things like running a toy car at 5 cm/s or pushing a button 5 times per second, and seeing how much they do in a second. This leads students first to the idea of rates over unit time intervals, which is the key idea behind how rates work. Once rates with unit time intervals are understood, it is easy to extend the experiments. Have students run the experiments again but with more time, and see what happens! Then, they will likely lead themselves to the D=RT equation in full generality themselves, combining both physical and inquiry-based learning. For more complicated systems (involving combining D=RT of multiple rates or multiple distances), students can again use physical experimentation to conjecture results and gain physical intuition for why things work. This could potentially be a way to implement the IMBI ideas within an elementary school context, helping students develop early math skills.

The next topic to discuss is one at the middle to early high school level, vectors. A full discussion of how to treat vectors would be far too extensive for this report, so we only discuss a small subsection of this. Vectors are incredibly useful mathematical tools, but their concept is hard to explain by itself. Why do we shove multiple pieces of information into one thing called a vector, and what does it represent? Why do
we define operations on them like the dot and cross products so...weirdly? When vectors are considered from a physical perspective, they make much more sense.

Let us have a person A, and let's say A walks somewhere. Well, that gives us two questions - how fast are they going, and in what direction are they going? Once we have this information, we can figure out where A is going and where they'll be at a given time, because we know their direction and speed. We can combine this information into one object, and call it a vector \([5, 6]\).

Now, what happens when the motion of A changes? Our next question can be "what happens when we put this person on a train?" With our current tools of only magnitude and direction, this is a hard question to answer: how can we add two things that don't point in the same direction? The answer is of course, components, as students can explore via hands-on physical demos and experiments, and hopefully propose a solution by themselves. Introducing the idea of components through this problem makes it clear why they are important—they allow us to answer the previous physical question that we didn't know how to answer beforehand. Notably, this also tells us how we add vectors: to get the velocity of the person relative to the ground, we simply add the person's velocity relative to the train with the velocity of the train relative to the ground \([5, 6]\).

Once again, this has introduced students to key ideas of vectors via a physical approach of experiment - playing with things they see in the real world and formulating larger ideas to gain and demonstrate competency with mathematics. The final topic to discuss in this treatise is one at the "capstone" level—the interplay between Quantum Mechanics and the introduction of principles of Linear Algebra in a high school curriculum. Quantum Mechanics, when introduced correctly, can help students understand the necessity for Linear Algebra and some of its rules.

In quantum mechanics, particles have certain properties when in a certain state—location, momentum, spin, and such—and the easiest way to combine all these properties is to put them into a state vector. Then, physicists will try to measure properties of particles using operators, and it turns out that these measurements are random, but also satisfy linearity, making them the linear operators on the state space. Two of the most important of these operators are the position operator and the momentum operator. However, it turns out that “regular” state space is easy to calculate positions in, but very difficult to calculate momentums in. The solution is to apply a Fourier Transform onto the Hilbert Space and operators that makes the position operator take a difficult-to-compute form and the momentum operator take a simple form. \[7\]

This of course, provides a motivation for introducing the idea of linear algebra. After introducing the ideas of quantum mechanics, ask students “how can we store all of this information concisely in one thing?” How can we operate on it and satisfy this linearity property? We look back at our discussion of vectors: we stored the direction and magnitude of the motion in an object that we called a vector. Similarly, we store all of these properties in another vector that we call an element of a vector space. We can then continue to extend these ideas to develop the ideas behind linear operators and why we need them, and then explore these ideas further.

This report has discussed physical learning and how to utilize it in a program like the IMBI Intiative. It has outlined examples of how to implement such an approach in curricula throughout the elementary to high school spectrum, but students could also do this independently or within other course subjects (such as physics courses). Overall, kinesthetic learning is a learning style that needs to be implemented in math education more, and the IMBI program should implement credentialing that interacts with this method.

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**Notes**

The topics discussed here most line up with M100d, 100f, 101a (d=rt). The other two topics, abstract linear algebra and vectors, do not correlate directly with any Math Badges.
Math is present in nearly every aspect of our lives. It permeates everything from our navigational systems to finances. It protects us from hackers and fraudsters. It advances our knowledge of chemistry, physics, and other sciences. Mathematics is the backbone of modern society; it is the basis upon which the sciences are built.

Yet many people conflate the idea of mathematics with advanced and complex formulas, or unintelligible symbols. So over the course of this internship, many examples of “nontraditional” mathematics have been collated to create a narrative that challenges the conventional public opinion of mathematics. By bringing in examples that use only simple arithmetic, or a branch of mathematics almost devoid of numbers, the results of this internship aim to show that math is not just abstract numbers and symbols, floating in free space. Furthermore, this internship attempts to subvert the stereotype of “useless” math by showing the applications of a wide breadth of mathematics.

To do this, three general observations have been made about the general applications of mathematics in modern society. First: mathematics, in most fields, is generally constrained to simple arithmetic and algebra. Second: mathematics is an incredibly wide field of study, with some of the most interesting fields completely missing numbers. Third: mathematics synergizes well with many other sciences, primarily with computer science, physics, and chemistry.

A common theme that everybody has likely noticed is that in many careers, complex mathematics has been exchanged for greater mathematical literacy. Of course, this is not true for all fields, but for many softer sciences, and fields such as statistics and accounting, the need to understand what exactly the numbers mean is paramount. For instance, the math involved in forensic accounting for both accounting and insurance fraud is generally quite simple, mainly consisting of simple arithmetic and data analysis. But in exchange for the general knowledge of advanced mathematics, these statisticians and accountants must have a strong understanding of the math and methods.

In fields where mathematical literacy is essential, the Math Badge M100 is a key set of skills to master and understand. Math Badge M100 covers the basics of interpreting, modeling, and problem-solving, at a similar level to what forensic accountants are familiar with. For example, a key tool in any forensic accountant's toolbox is ratio analysis. Ratio analysis is the inspection of the ratio of two key values on a balance sheet and remains effective because of standard business practices (Association of Certified Fraud Examiners, n.d.). When a company begins to sell more items, it would be natural to expect that company to begin increasing its stock as well, so forensic accountants expect to see a steady ratio between a company's sales and inventory. Similarly, the same reporting item on an income statement or balance sheet is often expected to contribute the same to the same statement, regardless of the reporting period, so seeing a reporting item contribute significantly relatively more or less during a reporting period is a red flag (Association of Certified Fraud Examiners, n.d.). However, a ratio analysis test is meaningless if a forensic accountant analyzes two random, or worse, unrelated variables. This is where the necessity of being able to interpret one's data is revealed. If an accountant were to analyze the ratio of total sales and number of car crashes in Florida, for example, even if there was a single reporting period where the ratio changed, the statistic would be useless because it compares two unrelated variables.

At first glance, the idea of math without numbers or variables is absurd. The very concept of math seems to hinge on the fact that one manipulates numerical values to obtain a final answer. However, many branches of math defy this misconception: logic, set theory, model theory, category theory, and especially graph theory. For the sake of making this point, the focus will be on graph theory. For background, the idea of graph theory is that there are nodes, also known as points, and edges, which connect said nodes (Haque, S. 2021). And that's it! No numbers, no variables. It's just nodes and edges. Due to its seemingly simple nature, it is common for people to believe that there can't possibly be any applications for such
an abstract branch of mathematics. However, it is because of its simple and abstract nature that graph theory is so versatile and applicable in many situations.

At the moment, there does not seem to be a Math Badge with corresponds with the skills required to effectively utilize graph theory. Despite its straightforward description, graph theory is actually utilized every day by nearly everybody, whether it be for driving or networking. Let us consider graph theory through the lens of networking. In this case, we could let each person be represented as a node, with edges connecting people who already know each other. So, to gain the most connections the quickest, one would seek out the node with the highest number of edges leading into it. Alternatively, graph theory has many applications on the road. Anybody who has used any navigational system has utilized graph theory via navigational systems (Ting et al., 2000). By representing different significant locations as nodes and the paths connecting them as edges, navigational systems use graph theory to find the most efficient path between your position and your destination. Furthermore, the graph theory is extensively used to simplify the process of programming traffic light patterns (Dowling, J., 2024). By representing each path a car can take as a node, and using edges to connect paths that can crash, a graph representing all possible crashes has been created. Then, by coloring each node such that no two connected nodes share a color, it can easily be determined which paths can go at once, simplifying the complex problem of cluttered or dangerous traffic intersections. These previous examples reflect not just the value of abstract mathematics, but also the fact that math without numbers can still be applicable.

It was noted at the beginning of this report that math is the basis on which all the sciences are based. It is then unsurprising that mathematics synergizes incredibly well in all sorts of fields, from chemistry to computer science. When students lament about the uselessness of mathematics, they tend to be shortsighted and unfamiliar with any professional field, especially the hard sciences and computer science. Even more notable are the “higher math courses,” such as multivariable calculus, number theory, or abstract algebra have definite uses in practical situations. For example, pathfinding algorithms are often integrated into graph theory on computers, number theory is the building block of internet security, and much of calculus is motivated by physics.

Similar to the point regarding abstract mathematics, there is no Math Badge that correlates with the general application of higher mathematics in practical situations due to the advanced nature of higher mathematics. The current problem with many math classes in the real world is their lack of motivation; it is rarely, if ever, mentioned within a classroom setting where and why one would find themselves applying mathematics. Perhaps introducing a motivation to certain key concepts, such as applications of integration or derivation would help not just create meaning in the math but also build an interest in mathematics in students. Let’s suppose that a beginning calculus student, currently learning about derivatives, wanted to learn about why this math was important. Well, the teacher could introduce a subtopic of derivatives: optimization, which is the process of finding the relative maximums and minimums of any given function (Dawkins, n.d.). The great thing about derivation and optimization is that optimization is not simply constrained to two-variable functions, but can even be extended to multivariate functions, so this motivation exists not just in introductory calculus courses, but in advanced calculus courses as well. Integration, the inverse of derivation, also has its uses in physics and statistics. For instance, to calculate the displacement of an object, one would simply integrate the time-velocity function over the given time interval. (Urone et al., 2024). Or to find the probability of certain events occurring and given a probability density function, integrating the function over certain intervals will produce the desired probabilities (Pennsylvania State University, n.d.). And even in number theory, many of its topics, from order to modular arithmetic happen to have major applications within computer science, from cryptosystems to random number generation (Arizona State University, n.d.; Micah, 2024). These examples, and the many more out in different fields of science show that there is a great amount of motivation to be found for pursuing higher mathematics.

Mathematics doesn’t have to be this mysterious machine that only exists in students’ lives for an hour a day. Instead, students should be encouraged to think about mathematics from different perspectives, whether it be from a programming, artistic, or statistical standpoint. Students should understand not just the math itself, but its applications and why they are learning mathematics. By encouraging students to
recognize that although elementary math is universally used, many branches of strange and interesting mathematics can play significant roles in their lives. Thus, the greatest goal a math class can achieve is providing a powerful motivation for students to pursue these topics with passion and interest.

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Applications and Education of Mathematics based on Real World Connections

by Roy Q. Wang, Illinois Mathematics and Science Academy

What is the probability that you sprain your ankle? On an incredibly rude estimation, the answer is 50%. The ankle is either sprained or remains healthy, but then, based on increasingly more influential factors, this probability may decrease to a much more acceptable probability: perhaps .005%. This is just one example of how mathematics may be modeled to showcase real-world problems. In other seemingly unrelated aspects of the real world that make up critical functions of global economics, mathematics is found as a dominating aspect. Even more, these mathematical patterns are beautiful examples for education in mathematics.

The stock market, a critical portion of the United States economy, is influenced by the mathematical concept of a random walk. In a random walk problem, someone constantly moves to the right while randomly moving up or down. This is reflected in the stock market due to an idea that predictions in the stock market are completely random. Therefore, the central limit theorem is consistently displayed in the behavior of the stock market, influencing decisions in call options and other derivative investments. The models predicting prices of call options is based upon beating the odds in a generated bell curve. This can be used to teach bell curves, probability, Pascal's triangle, and combinatorics, which falls under learning standard 113 and 213.

Just as relevant is communication in secret. Throughout time, nations have used different encryptions to prevent messages from ending up in the wrong hands. If each letter of the alphabet is expressed as an integer modulo 26, we get a fascinating real-world analogy to arithmetic and remainders. Many have heard of the Caesar cipher, and could attempt to explain it using mathematical reasoning and modular arithmetic. After this, the Vigenère cipher is a logical follow up. This cipher has a word as the key, acting as a Caesar cipher for each individual letter. For example, if the keyword was "ABCD," each individual alphabetical character would be a number from 1-26, and that value would be added to the phrase which was being encoded (in this case, 123412341234). This creates a Caesar-like cipher that has infinitely more possibilities.

In terms of mathematical formulas, if the keyword is repeated enough number of times so that the total length is equal to the length of the plaintext, for plaintext p1p2...pn, keyword k1k2...kn and ciphertext c1c2...cn, the encryption method is ci = (pi + ki) mod 26 for each i in total n, if n was the length of the plaintext. By this same logic, the decryption logic would be pi = (ci - ki) mod 26, which makes a potential algorithm for encryption or decryption very simple. This teaches not only arithmetic and remainders, but also functions with inputs and outputs, which falls under math badge 112.f, 100.f, and 101.a.

Communicating in secret is important to those who wish to remain anonymous. But, what if the goal is to bond over expressed emotions? Although relatively obscure, the math present in music defines all relationships between different notes and is key to music theory. For example, the frequencies of notes when jumping up octaves will double in each octave. Then, each of the 12 notes of the chromatic scale can be expressed as $2^{(x/12)}$, making each ratio consistent. In math education, this is a simple yet elegant way to teach ratios and exponents, reflecting math badges 202.a and M100.

However, just as interesting is the idea of the musical note: when visualized in 2 dimensions, it takes the form of a sine wave. Using the idea of sound passing through different frequencies, sinusoidal transformations and the creation of sinusoidal functions could be deeply intertwined with music, and allow children to understand a vital connection between sounds and waves. This would teach the standards of M205, and in more advanced calculus classes, the Fast Fourier Transform, the summation of many different sine waves that creates a comprehensive sound.
Humans not only need to communicate, but survive. One critical concern is global disease, or a pandemic, which can be seen in our recent COVID-19 pandemic. To learn more about these plagues, disease modeling takes many factors into account, such as time before symptoms, mode of spread, and rate of spread. Using mathematical models involving integrals, researchers have found the rate and probability of disease spread, which can also be seen as the area under a rate curve. For example, RIVM is a model to track the spread of COVID, which can be seen below.

When transformed back into a math classroom, this teaches area, probability, and rates, which include the math badges M113 and 151.g. Not only does an analytical approach to learning this work, but the creation of models to calculate rate and probability would strengthen the skills already learned and provide a feasible alternative to a written test.

The influence of mathematics in various facets of life, from finance to communication to public health, showcases the importance of learning math in connection to the real world. Many seemingly unrelated parts of life, such as prescription medicine, are influenced directly by mathematical models that can be taught in tandem with mathematical ideas. The integration of math with real-world applications will allow educators to reinforce knowledge and encourage mathematical exploration.
An Exploration of Mathematical Applications Outside of Traditional Classroom Settings

by Lucas Brower, Illinois Mathematics and Science Academy

Abstract

Mathematics intersects with many real-world applications, although only strictly traditional classroom diagnostic tests determine proficiency in specific mathematical topics. Math badges offer an alternative method to demonstrate proficiency in various subjects within math. This paper investigates computer graphics, technology, ray tracing, optimization functions, and convolutional neural networks, unveiling intricate connections between math and real-world applications. Utilizing the math badge system, the explorations will be assigned to a designated topic that best describes the type of math that each intersection covers. Synthesizing insights from these explorations demonstrates the profound impact of mathematical concepts in various domains outside the traditional classroom setting and highlights the versatility of mathematical principles. This versatility necessitates a method for students to exhibit adeptness in various ways, contributing to a more accessible form of education.

The Explored Intersections of Mathematics

Technology and Coding

Technology is integral to our daily lives, and math is fundamental to its design, development, and function. Coding is ubiquitous in the world, and without programming our lives would be significantly different as technology reliant on code allows us to commute at tremendous speeds, communicate with people in different cities in real time, and have access to nearly limitless amounts of information.

Coding is best described as the composition of sequences of instructions that computers can follow to perform designated tasks. Fundamentally, coding is the logic of a computer program (Mcgee, 2023). This is the first intersection with math, as logic is a branch of mathematics concerned with reasoning and argumentation, which allows for the construction of algorithms with the implementation of conditional statements (IF-ELSE) and logical operators (i.e., AND, OR, NOT). Expanding upon basic logic, coding incorporates discrete mathematics, which includes topics such as propositional logic, predicate logic, and set theory, which all provide the theoretical foundation for programming languages and computational thinking (Rapaport, 2013).

While computers are exceptionally fast, and they are only getting exponentially faster (Routely, 2017), the ultimate bottleneck to the speed of a program is the efficiency of the code. Analyzing the efficiency of algorithms requires math through complexity analysis and accurately modeling algorithms as functions. With Big O notation, programmers can utilize math to create functions reflective of the worst-case or average-case scenarios in terms of time taken (Vernon, 2005). This knowledge helps select appropriate algorithms and optimize code for better time or space complexities for given problems. Common model functions are linear or exponential correlations between input size, time usage, and memory allocation. Another type of analysis is asymptotic analysis, which deals with the behavior of functions as they approach certain limits. These types of analysis are used to describe the upper, lower, and tight bounds on the running time or space complexity of algorithms. Applying the asymptotic analysis to programs can identify desirable scalability characteristics, allowing the handling of larger input sizes efficiently. Ultimately, math can be used to quantify the efficiency of programs and qualitatively analyze algorithms.

Computer Graphics

Computer graphics are what generate the image on your screen. From video games and movies to virtual reality and computer-aided design (CAD), computer graphics represent what is occurring in the programs.
As with most things, the core of computer graphics lies in mathematical principles that enable the creation and rendering of 2D and 3D images on digital screens.

In Computer Graphics, objects are represented in a coordinate system, typically with a cartesian coordinate system in 2D, or a Cartesian/Cartesian-like coordinate system in 3D (Baker, 2024). Simple exercises in creating functions to display objects rendered in different coordinate systems (Polar, Cylindrical, etc...) could help offer alternative ways to understand the applications of concepts in class. Understanding how to interpret and manipulate coordinates is essential for placing objects accurately within a scene and contextualizing the usage of various coordinate systems.

Transformations involve altering the position, orientation, or scale of objects within a scene. These include translation, rotation, scaling, and shearing. Typically, transformations are represented by matrices, which are mathematical structures used to perform operations on vectors (Margalit, 2019). For example, translation involves shifting an object's position along the x, y, or z-axis by adding constant values to its coordinates. Rotation involves changing the orientation of an object around a specified axis by applying trigonometric functions to its coordinates (Multiplicative Rotational Matrices).

Rasterization is one method computers utilize to convert vector graphics into raster images. Vector graphics are one format for how geometries are stored and raster images are what you see on your screen (Indiana University, 2023). Scan conversion determines which pixels on the screen correspond to the geometric primitives being rasterized. Mathematically, this is the discretization of continuous geometric primitives into pixels. Each pixel is split into a grid, and if a polygon is included in a part of that grid, that part is considered to be part of the polygon. After scan conversion, the color and attributes of each pixel must be determined based on the properties of the geometric primitives. This may involve interpolating attributes such as color, texture coordinates, and surface normals across the primitive's surface using antialiasing. This can be achieved through linear interpolation or other interpolation techniques to calculate pixel values (Smith, 2023). Essentially, the color and texture attributes of each pixel are the summation of all the normalized color values of the parts of the grid.

**Ray Tracing**

Ray Tracing is the expansion of math within computer graphic techniques as an exemplar of Monte Carlo integration. Monte Carlo integration is utilized to approximate the integrals for rendering scenes with intricate lighting and shading effects (Agu, 2007). Using random sampling techniques, Monte Carlo integration efficiently calculates the contributions of light to each pixel in the final image. By accumulating these samples over multiple iterations, Monte Carlo integration produces high-quality images with accurate lighting that closely approximates the true value of the pixel colors.

Ray tracing traces a viewpoint back to the original light source, rather than from the light source to the viewpoint (University of Washington). The camera sends numerous amounts of rays from each pixel, each with a slightly different variance in its trajectory. Each ray attempts to reach a light source, reflecting or refracting off of solids until intersecting a light source. The ray stores the physical data of every surface it intersects with before reaching the light source. When a ray encounters a light source, it returns an RGB value calculated from the stored physical data which the pixel should assume.

The math behind ray tracing is found in the intersections with the geometric primitives rendered in an image. Determining the intersections of a vector and a plane involves understanding cross-products, normal vectors, and accurate distance formulas. Following the detection of intersection, the reflection and refraction of a light ray are determined by the surface that it intersects with. A specular reflection has a mirror reflection across the normal vector of the surface, while a diffusive reflection is the random refraction of a light ray (Benton, 2018). These different types of reflection types allow for translucent, reflective, or standard types of surface appearances.
**Convolutional Neural Networks**

Image processing through convolutional neural networks allows machines to autonomously extract information and make informed decisions based on the images provided (Simplilearn, 2023). The term convolution simply refers to two functions acting on each other to create a third behavior. Mathematically, in this context, convolution is represented as the element-wise multiplication of the filter with the input image followed by summation, producing a feature map that highlights relevant spatial information. The math “filters” that are applied to each node of the neural network generally have meaningless units that are solely multiplicative values meant to produce activation behavior. Each function could be any mathematical operation and nonlinear activation functions are required for nonlinear behavior. This behavior extracts features such as edges, textures, and patterns, or determines objects (Han, 2012).

Nonlinear activation functions, such as the rectified linear unit (ReLU) or sigmoid function transform the linear combination of input features into nonlinear mappings, enhancing the network’s capacity to model intricate patterns and nuances present in images.

Each parameter that these convolutions require obtains its optimum value through optimization (Ng, 2013). One common optimization technique is gradient descent. Gradient Descent iteratively updates parameter values in the steepest descent of the objective function. The gradient is a matrix, or vector, of the derivatives of each variable within the function. The directional derivative of a function is the dot product of the unit vector (direction) with the gradient, meaning that the greatest ascent/descent is parallel to the direction of the gradient ($a \cdot b = |a||b|\cos \theta, \theta = 2k\pi$ s.t. $k \in \mathbb{Z}$).

Specifically for CNNs, or NNs in general, backpropagation reduces computation per iteration of gradient descent. Backpropagation utilizes the dependence that nodes later on in feed-forward neural networks have on the previous nodes to calculate the error fewer times per complete iteration than traditional gradient descent through iteratively processing a dataset of training tuples by comparing the network’s predictions with the known target values (Han, 2012). With each training tuple, the algorithm adjusts the weights to minimize the mean-squared error between the network’s predictions and the actual target values. This adjustment occurs in a backward direction, starting from the output layer and propagating through each hidden layer to the first hidden layer, hence the name “backpropagation.”

**Data Visualization**

When considering large data sets, the most important step to understanding the set is accurate data visualization. Data and information visualization is the practice of designing and creating visualizations that are easy to understand and represent a large amount of complex quantitative and qualitative data and information through either static, dynamic, or interactive visual tools. More generally, these visualizations are intended for a broad audience to help them visually explore and discover important insights into otherwise subtle or difficult-to-identify correlations and trends. This field is important and is prevalent within research from computer science, psychology, business methods, financial data analysis, and market studies, to manufacturing production control or drug discovery.

Data visualization intersects with statistical analysis. Measures such as mean, median, mode, and standard deviation are essential for summarizing data and identifying trends (IBM, n.d.). Visual representations like histograms and box plots rely on these statistical measures. Mathematical models like correlation coefficients and regression analysis help in understanding the relationship between variables, which can be visualized through scatter plots and trend lines.

Dimensionality reduction techniques like Principal Component Analysis (PCA) and t-distributed Stochastic Neighbor Embedding (t-SNE) use mathematical algorithms to reduce the number of variables while preserving essential information (Sharma, 2024). These techniques are crucial in visualizing high-dimensional data in lower-dimensional spaces, making it easier to explore and interpret complex datasets. Understanding the mathematical principles behind dimensionality reduction helps in selecting appropriate techniques and interpreting visualizations effectively. In classroom settings, introducing dimensionality reduction concepts alongside data visualization provides students with advanced analytical tools and prepares them for handling large-scale datasets in various fields such as machine learning and data science.
Classification of Intersections with Math Badge M212

The intersections with mathematics within these explorations cover vast topics, with minimal overlap in the main topics. Despite this, M212 addresses topics from three explorations: modeling with CNNs, dimensionality reduction in data visualization, and interpreting error values when relating data to models. The badge focuses on modeling, predictive analytics, and data-driven decision-making. This includes the full modeling cycle, constructing and refining statistical questions, selecting appropriate predictive models, and understanding the interplay between data and models. After initially understanding modeling techniques, machine learning techniques, optimization algorithms, and error analysis are utilized to improve predictive model fit and enhance decision-making processes. Additionally, the badge emphasizes critical thinking in evaluating correlations versus causations, identifying confounding variables, and extrapolating model limitations. Overall, it equips students with the necessary tools to synthesize mathematical understanding and apply investigative methods effectively in diverse real-world contexts regarding data analysis.

Badge Indicator Classifications

(212.a) Engaging in the Model Cycle
This indicator envelopes the modeling cycle holistically, including determining the problem, choosing a strategy, finding values, explaining what the values mean, checking that the values make sense, revising, and sharing findings. The best correlation between these objectives and the explorations is the gradient descent optimization method. This method involves selecting an objective function to optimize, which, in this case, would be the model of the data set. Finding the best fit for a specific model would be applying the optimization algorithm to the model parameters, and then using these optimal values would allow statistical analysis to be applied to determine the viability of the model type. For example, the R2 correlation value for linear regression models determines the statistical correlation of the model to the data.

(212.b) Constructing Questions for Predictive Modeling
The 212.b indicator concerns the differentiation between statistical and non-statistical questions, creating statistical questions suitable for predictive modeling, suggesting statistical questions to produce a given data model, and identifying relationships between variables. This particular indicator best correlates to data visualization. High-quality visuals of data facilitate trend recognition and assist in creating statistical questions for predictive modeling that include variance. Data visualization would easily and quickly eliminate questions that are not statistical (i.e. do not include variance). Dimensionality compression shows relationships between specific variables and is used to identify these relationships amidst the presence of other variables within a model.

(212.c) Selecting Predictive Models To Answer Statistical Questions
Similar to 212.b, this learning objective fits with data visualization. It includes selecting appropriate predictive models, and determining if data appears to fit a model. This means that in group models the visualization would show a cluster of data points about specific axial locations, whereas a regression model would visually look like a general function along an axis. By understanding how to properly illustrate the trends of sample data points, data visualization demonstrates proficiency in understanding statistical questions and relationships between variables.

(212.d) Understanding Data = Model + Error
This learning objective covers using a statistical model as a function to predict a score, differentiating between the empty and complex models, showing how to represent a linear relationship with least squares regression, and understanding that a perfect prediction does not usually exist by recognizing errors visually. A combination of gradient descent setup and data visualization techniques cover these objectives. The gradient descent technique also utilizes an error function to quantify the accuracy of
the model function. Understanding how to set up this error function demonstrates the capability of using a statistical model to predict a score and showing how to represent a linear relationship with least squares regression given that this equation is chosen. Data visualization would be used to prove that a perfect prediction does not exist by drawing the best model and displaying the variance within statistical questions.

(212.e) Implementing Machine Learning Models
Utilizing convolutional neural networks within image processing encompasses this objective completely. Given that convolutions are linear functions, the convolutional model is a linear model to fit data with one or more predictors. Altering some nodes to be classification methods, the model would then utilize at least one type of classification method to make a prediction. Similar to the classification, altering the architecture of the CNN slightly would produce a clustering algorithm to make algorithm. This alteration would involve replacing the convolutions within each node with classification methods so the CNN would cluster the data points to different classifications within the sample dimensional space.

(212.f) Improving Predictive Model Fit
This indicator also closely aligns with the initializing of gradient descent algorithms and data visualization. To properly assign objective functions, it’s necessary to understand that some models visually appear to fit data better. Data visualization techniques like dimensionality reduction can both decrease the number of variables in a predictive model and visually depict the better fit that some models have. Finally, after applying the gradient descent algorithm to the parameters of a model, simply running the model with additional functions would allow for a new model to be created with transformed variables.

(212.g) Using Predictive Models To Make Predictions
Following the application of gradient descent, or some other optimization algorithm, to a predictive model, either graphing or selecting a specific point to input values to the predictive models would obtain a prediction. Specifically for gradient descent, without understanding the output of a predictive model, the error function cannot correctly quantify the accuracy of the model and will fail to traverse the gradient to an optimal fit. Finally, without listing the limitations of a predictive model, such as the least possible error, a gradient descent algorithm could potentially fall into infinite recursion as it attempts to reach a certain threshold of error.

References


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Math is used outside of school much more than the average student might think. Every little thing in the world is determined with math. When students think of math in the outside world they may think electricity or carpentry or even cooking, but not theater. Backstage, lighting for a theater has a lot going on that can be directly taken from math learned in the classroom. To stay safe, new math can be necessary to learn. Yet the math a student has already learned can be generously applied to things like design concepts. All sorts of problems can be solved just using math. Lighting has a whole world of math surrounding it that is overlooked. Anything learned in stage lighting can be applied elsewhere, and often vice versa. Giving kids opportunities to use math in the real world in fun situations like lighting design will make them like school more. Helping youth find a love for math outside of the classroom should be a priority.

Expanding Knowledge
Math is an important part of everyone’s life in the sense that safety is constructed by math. Staying safe in the world is essential and math has been doing that job from the beginning. For example: when people map out cities and roads, that math not only helps the world run smoothly but also keeps everyone safe from things like collisions or natural disasters. Or the math that goes into building metal detectors for airports is designated to keep people safe. Building things like this uses math for real world situations. Regulations like American voltage being around 115-120 volts are so important to everyone’s day to day life and nobody even knows it. Learning about all the little things like this when going into adulthood and owning a home or traveling can be good to know but also vital for certain occupations. In stage lighting, the difference between knowing the amperage of a cable and not knowing can be life or death.

Oftentimes people do not consider things in the real world as math because they don’t look like the equations they had to solve in class. While the math isn’t obviously displayed outside of school, it is used in more discrete ways. For example: algebraic equations with numbers and letters are not usually seen in some random job, however they can still be applied. Say someone is trying to determine how long it will take to recover from an operation. The operation, how old the person is, what part of the body the operation is affecting and so many other variables will equate to recovery time or other outputs like amount of pain. Thinking about this and purposefully applying it is a way students could use the badge 104.i with variables from statistics. Inserting variables is a constant form of math in many parts of life that come naturally and subconsciously to most. Putting in time to learn these attributes is a way to expand previously learned knowledge and apply them outside of the classroom. Not all math has to come from school, and recognizing that can better aid students in their futures as adults.

Applying Math
The world is full of creativity. Being creative doesn’t mean improvisation, in fact it can be very calculated. Designs for companies are not pieced together at random. People go to school and train to do what they do. Most things are purposeful. It can take years to get something right, but with the right math the process can be quicker. The time it takes to find the perfect thing might not be time spent changing designs but actually tweaking math. Either way, math is being applied to better designs. The creations people make are deep-seated in math.

The math learned in school can be directly taught for certain real world situations, or it can be taught as a requirement and then applied to circumstances. In lighting design, lots of math is used and often it’s basic math that people already know. Geometry is a big part of lighting, and not just stage lighting. A good theater is very uniform and exact. Its lights should hit the same from both sides and have congruent angles.
At Charleston High School in Charleston Illinois they have a theater. Swickard Auditorium, based inside of the high school, holds 32 lights. There is no official lighting plot to reference for anything relating to the lights. A way that students could use math learned in a geometry class outside of the classroom would be to find the information needed for a lighting plot. A clinometer can be used to find the angle of each light. Measuring tapes or other tools can be used to find the radius of displayed light and coordinates. Understanding angles and units of measurement learned in the classroom can easily be applied in stage lighting.

Taking in measurements and putting them into a lighting plot for Swickard Auditorium will show that it is a very unorganized theater. Not a single light hits the stage at a 45 degree angle. According to the McCandless theory, the best angle for lights to hit a stage is at a 45 degree angle. Not only do the lights hit at random angles, they are also not congruent to the light opposite the stage of them. Light number 30 is opposite light number 24 on the stage. 30 barely hits the wall and covers mostly curtain space while 24 fully hits the wall and floor. 30 is also at a 78 degree angle while 24 is at 71 degree angle, which makes a lot of visible difference.

When designing a theater, the simplest, most uniform and correct way to build it is for the lights opposite each other to be exactly the same. This gives unity to the stage and actors on it and is simpler for the lighting plot measurements. A student could easily take the blueprints of Swickard Auditorium and make a lighting plot solely based on the building. Then use that lighting plot to map out even and congruent lights for the theater. Making a plot like this would be a perfect example of badge 151.d. It is so simple and easy to do things right with lighting design. Irregular designs make lighting difficult, however even in a theater like Swickard Auditorium, math can be used to solve any problem.

**Math Opportunities**

Students often feel like their classes are not preparing them for life outside of high school well enough. This is because they have never learned how things like math apply to the real world. The skills a math class gives students can be applied to any situation. Working with IMBI or just math outside of class in general helps students feel more prepared for college and adult life. Knowing they can use things they learned in class in the real world makes them feel smarter and retain the things they learned better. Making students aware of how things relate outside of school paves the path for more motivation in math classes.

It is important for students to enjoy academics and school. They spend the majority of their youth learning and it is easy to be discouraged from enjoying school when growing up in a rural town. Some people have very little opportunities in high school to further their education. Others are lucky and get to go to specialized academies. Students from both schools deserve to love math. It does not take coming from a rich school to go far with math. Encouraging students to do as much as they can is meaningful even if they can't do as much as other students at that point in time. No matter the resources, students should, at least, feel compelled to continue their dreams with math.

Anyone can do stage lighting, whether they grew up with opportunities to work with anything theater related or not. The things learned from stage lighting like drafting plots and working with electricity can be used as other things in life. Drafting lighting plots can help with pre-writing strategies and design skills. The skills students can get from doing math outside of the classroom are just as meaningful as those learned in school. And anyone can do it. Growing up in a rural town without theater or anything remotely like it doesn't mean anything. Just simple math and motivation can get people anywhere. Holding out and working hard, no matter the background, will lead people to achievement.

**Significance**

Helping kids like math is simple. It is so often displayed in a space outside of school that understanding the opportunities math gives students is highly beneficial. Applying math to real world situations is easier than people may think. Simple equations can be found all throughout the world. Learning math is often a necessary step for certain jobs. However, using math previously learned from school is also
very relevant. No matter where a student learns from, they will continue to use it for the rest of their lives. Math is everywhere and helping students comprehend that is essential to keeping a spark in their lives surrounding academics. Life can be unfair and people are not always given the same opportunities. But everyone deserves to be given the chance to find a love for math. Knowing that an unfortunate student can achieve the great things a more fortunate one can is important for students to know. Keeping a positive attitude towards learning and math will lead any student to great things.
Solving a Real Life Problem

By Nathan Ola

Math is everywhere in life, not just in school. It helps with understanding things around the world, like art, music, and sports. But sometimes, learning math in class can be hard for some people. Luckily, there are other ways to teach math that are more fun and easier to understand. By exploring how math is used in different parts of life and trying new ways to teach it, it can benefit more people to learn and enjoy math. After a total of five weeks of research into how math plays into both the small and big subjects in life, it can be concluded that math can be taught outside of a typical curriculum.

Although some students have no problem with learning math concepts in the traditional classroom, others find it difficult to grasp concepts. Traditional math classrooms often focus on memorization and abstract problem-solving, which can be challenging for many students. Alternative methods, like hands-on projects and real-world applications, offer a more engaging way to learn math. By using practical examples and collaborative activities, educators can help students understand math concepts better and develop critical thinking skills.

The first topic that can be a great asset in teaching math outside the typical curriculum is computer-aided design, or CAD. Computer-aided design (CAD) is a technology that has revolutionized the way products are designed and manufactured in the modern world. CAD software allows engineers, architects, and designers to create detailed and precise digital models of objects, buildings, and systems. These digital models can then be manipulated, analyzed, and tested before physical prototypes are ever produced, saving time and resources in the design process. CAD is extensively used in industries such as engineering, automotive, aerospace, architecture, and manufacturing, where accuracy and efficiency are paramount.

In the realm of education, CAD offers a powerful tool for teaching mathematics in a practical and engaging manner. By integrating CAD software into the curriculum, educators can provide students with hands-on experience in applying mathematical concepts to real-world design challenges. For example, students can use CAD to explore geometric principles by creating and manipulating 3D shapes, or they can apply algebraic equations to design mechanical systems with specified dimensions and constraints.

CAD provides a rich environment for teaching a variety of mathematical concepts. Geometry, for instance, can be taught through CAD by exploring properties of 2D and 3D shapes. Students can construct polygons, such as triangles, quadrilaterals, and hexagons, and investigate their angles, side lengths, and symmetry using CAD software. Additionally, CAD enables students to delve into spatial relationships by creating and manipulating 3D objects like cubes, spheres, and pyramids, while exploring concepts such as volume, surface area, and congruence. Furthermore, CAD facilitates the exploration of transformations, such as translations, rotations, and reflections, allowing students to visualize the effects of mathematical operations on geometric figures in real-time. Through hands-on experimentation and exploration, students can deepen their understanding of mathematical concepts and develop problem-solving skills within the context of design and engineering.

The next topic that can be used to teach math skills is food, specifically the processes and health surrounding food. Cooking provides a practical and engaging context for teaching various mathematical concepts. From measuring ingredients to adjusting recipe proportions, cooking involves a range of mathematical operations that can enhance students’ understanding of fractions, proportions, and arithmetic. For example, students can learn about fractions by measuring out ingredients like flour, sugar, and liquids using measuring cups and spoons. They can also practice multiplication and division when scaling recipes up or down to adjust serving sizes or quantities. Additionally, cooking can introduce concepts of time and temperature, as students follow recipes with specified cooking times and oven temperatures, and learn to convert between different units of measurement such as ounces to grams or Fahrenheit to Celsius.
Still touching on the subject of food, tracking macros, or macronutrients, offers another avenue for teaching math in a practical context. Macronutrients, which include carbohydrates, proteins, and fats, are essential components of a balanced diet, and tracking their intake can help individuals achieve specific health and fitness goals. Students can learn about percentages and ratios by calculating the proportion of each macronutrient in a meal or snack based on its nutritional information. For instance, they can calculate the percentage of calories from carbohydrates, proteins, and fats in a serving of pasta with marinara sauce and grilled chicken breast. This process not only reinforces mathematical concepts but also promotes awareness of nutrition and healthy eating habits.

To calculate the appropriate amount of calories and macros for a meal or diet plan, students can apply mathematical formulas and principles of nutrition. First, they can determine their daily caloric needs using equations based on factors such as age, gender, weight, height, and activity level. Then, they can allocate these calories into specific percentages for carbohydrates, proteins, and fats based on recommended dietary guidelines or personal preferences. For example, if someone needs 2000 calories per day and wants to follow a balanced diet with 50% carbohydrates, 25% proteins, and 25% fats, they would aim for 1000 calories from carbohydrates, 500 calories from proteins, and 500 calories from fats. By applying mathematical reasoning to nutritional planning, students can develop a deeper understanding of the relationship between math, food, and health.

The third topic that can teach math concepts is nature. Nature serves as a captivating and immersive classroom for exploring mathematical concepts, offering endless opportunities for observation and discovery. The golden ratio, a mathematical proportion often found in natural patterns, can be observed in the arrangement of petals in flowers, the spirals of seashells, and the branching of trees. By studying these natural phenomena, students can develop an intuitive understanding of the golden ratio and its aesthetic significance in art, architecture, and design. Moreover, symmetry, another fundamental concept in mathematics, is prevalent throughout the natural world, from the bilateral symmetry of animals to the radial symmetry of flowers and snowflakes. By examining symmetrical patterns in nature, students can grasp the concept of symmetry and explore its applications in geometry and visual arts.

Adding on, fractals, intricate geometric shapes characterized by self-similar patterns at different scales, abound in nature and offer a fascinating window into mathematical complexity. From the branching of trees to the coastline of a rugged shoreline, fractal patterns emerge in diverse natural structures. Through hands-on exploration and visualization, students can understand the recursive nature of fractals and explore mathematical concepts such as iteration, recursion, and infinity. Furthermore, the mathematical constant pi, which represents the ratio of a circle's circumference to its diameter, is embedded in natural phenomena such as the orbits of planets, the ripples on a pond's surface, and the shapes of clouds. By observing and measuring these natural occurrences, students can appreciate the universality and beauty of mathematical principles in the world around them, fostering a deeper connection between mathematics and the natural sciences.

The fourth topic that can be used to teach math skills is photography. Photography provides a compelling avenue for integrating mathematics into visual arts education, offering a platform for exploring geometric principles, proportions, and algorithms. Through the lens of a camera, students can engage with mathematical concepts such as framing, composition, and perspective. For instance, the rule of thirds, a fundamental principle in photography, divides an image into a grid of nine equal sections, with key elements ideally placed along the grid lines or their intersections. By applying this rule, students can develop an understanding of proportions and spatial relationships, as well as the aesthetic balance between foreground and background elements. Moreover, concepts like focal length and aperture, which determine depth of field and focus in photography, involve mathematical calculations and algorithms. By experimenting with different camera settings and observing their effects on image clarity and depth, students can explore mathematical concepts such as ratios, proportions, and geometric optics in a tangible and visually engaging manner.

Furthermore, digital image processing offers a wealth of opportunities for teaching mathematical algorithms and computational concepts through photography. Students can learn about pixel
manipulation, color theory, and image enhancement techniques using software like Adobe Photoshop. Algorithms such as edge detection, histogram equalization, and Fourier transforms, which are fundamental to image processing, provide a bridge between mathematics and digital art. By applying these algorithms to their photographs, students can gain insights into mathematical concepts such as convolution, frequency domain analysis, and spatial filtering, while also honing their problem-solving and critical thinking skills. Through photography, students can not only explore the intersection of mathematics and visual arts but also develop a deeper appreciation for the mathematical principles underlying digital imaging technologies.

In conclusion, CAD, food, nature, and photography offer diverse and immersive avenues for teaching mathematics beyond the traditional classroom setting. Through CAD, students can apply mathematical concepts to real-world design challenges, fostering creativity and problem-solving skills. Similarly, exploring mathematical patterns in food, nature, and photography allows students to engage with mathematical principles in tangible and meaningful ways, from the golden ratio in flower petals to the rule of thirds in photography composition. These hands-on experiences not only make math more accessible and relatable but also cultivate a deeper appreciation for the role of mathematics in everyday life. By embracing alternative approaches to math education, educators can inspire curiosity, creativity, and critical thinking in their students, empowering them to become confident problem solvers and lifelong learners.